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# A Model Based Fault Detection Scheme for Nonlinear Multivariable Discrete-time Systems

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**Abstract**— In this paper, a novel robust scheme is developed for detecting faults in nonlinear discrete time multi-input and multi-output systems in contrast with the available schemes that are developed in continuous-time. Both state and output faults are addressed by considering separate time profiles. The faults, which could be incipient or abrupt, are modeled using input and output signals of the system. By using nonlinear estimation techniques, the discrete-time system is monitored online. Once a fault is detected, its dynamics are characterized using an online approximator. A stable parameter update law is developed for the online approximator scheme in discrete-time. The robustness, sensitivity, and performance of the fault detection scheme are demonstrated mathematically. Finally, a Continuous Stir Tank Reactor (CSTR) is used as a simulation example to illustrate the performance of the fault detection scheme.

**Keywords**—fault detection, multivariable system, nonlinear discrete time system, adaptive estimation

## I. INTRODUCTION

In the model based fault detection approach; a model of the actual nonlinear system is obtained. Then the residual is generated by comparing the model response with that of the actual system output. In the presence of any fault, the system behavior will deviate making the residuals to grow. Once the residual exceeds a predetermined threshold, a fault is detected and declared active. In some of the previous works [1-2], the fault detection techniques were developed by considering a linear representation of the nonlinear system. Later in the last decade, some fault detection schemes [3] were developed for nonlinear systems by considering linear actuator faults rather than nonlinear faults. Since the generation of residual and the selection of threshold is vital for fault detection thus ensuring robustness of the fault detection scheme, a systematic mathematical procedure has been developed for the selection of threshold [5-8]. In addition to fault detection, the detected fault is identified (fault isolation) and also accommodated (fault accommodation). However, literature on various fault isolation and accommodation schemes could be found elsewhere [7-8].

It is important to notice that a continuous-time algorithm cannot be directly converted and implemented in discrete-time. In addition, it's very hard to show the stability mathematically due to the quadratic nature of the Lyapunov function. This issue has prevented many researchers to focus on the

development of detection and accommodation schemes for nonlinear discrete time systems.

Recently, fault detection schemes for both linear and nonlinear discrete time system were being developed for actuator faults [10-11], but their stability is guaranteed only when PE (persistency of excitation) condition is satisfied. Due to limited literature for discrete-time systems, in our previous work, we developed fault detection scheme for nonlinear discrete time systems with complex faults by relaxing PE condition, and with states fully measured [12] for a single input and single output system.

By contrast, in this paper, a novel fault detection scheme is developed for a class of nonlinear multi-input and multi-output discrete time systems with nonlinear incipient and abrupt faults occurring both in the states and the output. As the input and output signals applied to the nonlinear discrete-time systems are considered available, the faults are modeled as a nonlinear function of the input and output, and also could be occurring simultaneously or independently. Additionally, the faults could evolve at different rates while the time profile of the fault is still modeled by using exponential functions. A nonlinear estimator scheme with the online function approximation approach in discrete-time (OLAD) [12] is used not only to monitor the nonlinear discrete-time system but also to capture the fault characteristics. The developed fault detection scheme is tuned by using novel parameter update laws in discrete-time. Extensive analytical results are derived to show the sensitivity, robustness, and performance of the fault detection scheme.

## II. PROBLEM STATEMENT

Consider a nonlinear discrete time multi-input and multi-output dynamical system described by

$$\begin{aligned} x(k+1) &= Ax(k) + \varphi_s(y(k), u(k)) + \eta_s(x(k), u(k), k) \\ &\quad + \Pi_s(k - T_s)f_s(y(k), u(k)) \\ y(k) &= Cx(k) + \eta_y(x(k), u(k), k) + \Pi_y(k - T_y)f_y(u(k)) \end{aligned} \quad (1)$$

where  $x \in \mathfrak{R}^n$  is the state vector,  $u \in \mathfrak{R}^m$  is the input to the system,  $y \in \mathfrak{R}^p$  is the measurable output of the system,  $\varphi_s : \mathfrak{R}^p \times \mathfrak{R}^m \rightarrow \mathfrak{R}^n$ ,  $\eta_s : \mathfrak{R}^n \times \mathfrak{R}^m \times \mathfrak{R}^+ \rightarrow \mathfrak{R}^n$ ,  $f_s : \mathfrak{R}^n \times \mathfrak{R}^m \rightarrow \mathfrak{R}^n$ ,  $\eta_y : \mathfrak{R}^n \times \mathfrak{R}^m \times \mathfrak{R}^+ \rightarrow \mathfrak{R}^p$ ,  $f_y : \mathfrak{R}^m \rightarrow \mathfrak{R}^p$  are

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smooth vector fields whereas  $A \in \mathbb{R}^{n \times n}$ ,  $C \in \mathbb{R}^{p \times n}$  are known matrices. The nonlinear function  $\eta_s(x(k), u(k), k)$  represents the modeling uncertainties and disturbances whereas  $\eta_y(x(k), u(k), k)$  represents the sensor modeling uncertainties and noise. The nonlinear function  $f_s(y(k), u(k))$  represents the evolution of the nonlinear state fault dynamics and is modeled in terms of the measurable inputs and outputs, whereas  $f_y(u(k))$  represents the nonlinear output fault modeled in terms of the input. The diagonal matrices  $\Pi_s \in \mathbb{R}^{n \times n}$  and  $\Pi_y \in \mathbb{R}^{p \times p}$  denote the time profiles of the state and output faults which are given by

$$\Pi_s(k - T_s) = \text{diag}(\Omega_{s1}(k - T_s), \Omega_{s2}(k - T_s), \dots, \Omega_{sn}(k - T_s))$$

$$\Pi_y(k - T_y) = \text{diag}(\Omega_{y1}(k - T_y), \Omega_{y2}(k - T_y), \dots, \Omega_{yp}(k - T_y))$$

where

$$\Omega_{si}(k - T_s) = \begin{cases} 0 & \text{if } k < T_s \\ 1 - e^{-\kappa_{si}(k - T_s)} & \text{if } k \geq T_s \end{cases} \quad i=1, 2, \dots, n$$

and

$$\Omega_{yj}(k - T_y) = \begin{cases} 0 & \text{if } k < T_y \\ 1 - e^{-\kappa_{yj}(k - T_y)} & \text{if } k \geq T_y \end{cases} \quad j=1, 2, \dots, p$$

where  $\kappa_{si} > 0$  and  $\kappa_{yj} > 0$  are unknown constants that represent the rate at which the fault in the state  $x_i$  and in output  $y_j$  evolves. For small values of  $\kappa_{si}$  and  $\kappa_{yj}$ , these terms describe incipient fault whereas for large values these represent abrupt faults. Also,  $T_s$  and  $T_y$  denote the unknown time of occurrence of state and output faults, respectively. It is assumed that the initial system state is available i.e.  $x(0) = x_0$  and also the pair  $(A, C)$  is observable. Other assumptions include:

**Assumption 1:** the modeling uncertainty is unstructured and bounded [8, 9] i.e.

$$|\eta_s(x(k), u(k), k)| \leq \tilde{\eta}_s, \quad \forall (x, u, k) \in X \times \Omega \times \mathbf{T}$$

$$|\eta_y(x(k), u(k), k)| \leq \tilde{\eta}_y, \quad \forall (x, u, k) \in X \times \Omega \times \mathbf{T}$$

where  $X \subset \mathbb{R}^n$ ,  $\Omega \subset \mathbb{R}^m$  are the state and control input regions of interest, respectively, and  $\mathbf{T} \subset \mathbb{R}^+$  is the time interval prior to the occurrence of either state or sensor fault, i.e.,  $\mathbf{T} = [0, \min(T_s, T_y)]$  [6].

**Assumption 2:** The state and the input vectors are considered bounded prior to and after the fault occurrence. This assumption is a standard assumption in the literature [6].

Next define the nominal dynamics of the system as

$$x(k+1) = Ax(k) + \varphi_s(y(k), u(k))$$

$$y(k) = Cx(k)$$

In this paper, both the incipient and abrupt faults evolving in the states and the outputs are addressed. The representation given in (1) provides a general framework for a wide range of multi-input multi-output nonlinear systems with both state and output faults. Next we present the fault detection scheme for the system under consideration.

### III. FAULT DETECTION FRAMEWORK

The following nonlinear estimator is used for monitoring the multi-input multi-output system with fault in (1) and it is given by

$$\hat{x}(k+1) = A\hat{x}(k) + \varphi_s(y(k), u(k)) + K(y(k) - \hat{y}(k)) + \hat{f}_s(y(k), u(k); \hat{\theta}_s(k))$$

$$\hat{y}(k) = C\hat{x}(k) + \hat{f}_y(u(k); \hat{\theta}_y(k)) \quad (2)$$

where  $\hat{x} \in \mathbb{R}^n$  is the estimated state vector,  $\hat{f}_s: \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^l \rightarrow \mathbb{R}^n$  and  $\hat{f}_y: \mathbb{R}^m \times \mathbb{R}^l \rightarrow \mathbb{R}^p$  are the online function approximators in discrete-time (OLAD) with  $\hat{\theta}_s \in \mathbb{R}^l$  and  $\hat{\theta}_y \in \mathbb{R}^h$  are the set of adjustable parameters, and

$K \in \mathbb{R}^{n \times p}$  is a design constant matrix, which is chosen such that the matrix  $A - KC$  has all its eigenvalues within the unit disc [13]. In this paper, we consider a general class of online approximators. The OLAD schemes can be chosen to be neural networks, polynomial basis functions and so on whereas their parameters are tuned online with an adaptive update law. Many published works [4, 7-8] discuss various online approximator schemes and therefore omitted. The initial parameters of the OLAD is chosen such that  $\hat{\theta}_s(0) = \hat{\theta}_{s0}$ , so that  $\hat{f}_s(y, u, \hat{\theta}_{s0}) = 0$  and  $\hat{\theta}_y(0) = \hat{\theta}_{y0}$ , such that  $\hat{f}_y(u, \hat{\theta}_{y0}) = 0$  for all  $y \in \mathcal{Y}$  and  $u \in \mathcal{U}$ , where  $\mathcal{U}$  and  $\mathcal{Y}$  define the admissible range of input and output. The robustness of the OLAD scheme prior and after the occurrence of the fault is shown mathematically in the next section. Hence prior to the fault, the OLAD scheme does not update its parameters.

Define state estimation error as  $e_s = x - \hat{x}$  and output estimation error as  $e_y = y - \hat{y}$ . Under the ideal conditions with no modeling errors, a fault is declared active only whenever the norm of the output estimation error is greater than zero and subsequently when the online approximators ( $\hat{f}_s(y(k), u(k); \hat{\theta}_s(k))$  and  $\hat{f}_y(u(k); \hat{\theta}_y(k))$ ) become nonzero. In order to avoid false alarms due to unmodeled dynamics, the proposed fault detection scheme utilizes a dead zone to improve the robustness as

$$D[e_y(k)] = \begin{cases} 0, & \text{if } \|e_y(k)\| \leq \varepsilon \\ e_y(k), & \text{if } \|e_y(k)\| > \varepsilon \end{cases}$$

where  $\varepsilon$  is the threshold value obtained in the next section. A fault is declared active only if the output estimation error exceeds the predefined threshold. The systematic procedure for the selection of threshold is derived in terms of the modeling

error bounds viz.  $\tilde{\eta}_s$  and  $\tilde{\eta}_y$ , as presented in the next section.

By selecting a dead zone based fault detection similar to continuous-time, which is absent in other model based schemes [2, 3], it is possible to reduce missed and false alarms. However, the analytical proof is exceedingly complex unlike in the case of continuous-time. Additionally, the parameter update laws, to be introduced next, are not in any way similar to the continuous-time fault detection [3-8] or utilized in discrete-time control [13]. The update laws are derived for the purpose of detection.

The online approximator schemes  $\hat{f}_s$  and  $\hat{f}_y$  respectively are tuned using the following update laws

$$\begin{aligned}\hat{\theta}_s(k+1) &= \hat{\theta}_s(k) + \alpha_s \left( Z_s D[e_y(k)] - \gamma_s \|I - \alpha_s Z_s Z_s^T\| \hat{\theta}_s(k) \right) \\ \hat{\theta}_y(k+1) &= \hat{\theta}_y(k) + \alpha_y \left( Z_y D[e_y(k)] - \gamma_y \|I - \alpha_y Z_y Z_y^T\| \hat{\theta}_y(k) \right)\end{aligned}\quad (3)$$

where  $\alpha_s > 0$  and  $\alpha_y > 0$  are the learning rate or adaptation gains,  $0 < \gamma_s < 1$  and  $0 < \gamma_y < 1$  are the design parameters and  $Z_s$  is  $l \times n$  and  $Z_y$  is a  $h \times n$  matrix

$$\text{defined as } Z_s = \left[ \frac{\partial \hat{f}_s(y, u, \hat{\theta}_s)}{\partial \hat{\theta}_s} \right]^T \text{ and } Z_y = \left[ \frac{\partial \hat{f}_y(u, \hat{\theta}_y)}{\partial \hat{\theta}_y} \right]^T \quad (4)$$

It is important to note that the above proposed parameter update laws relax the critical requirement of the persistency of excitation condition (PE) for non-ideal cases i.e. system with modeling and approximation errors. Similar to the projection algorithm in continuous-time, the above proposed parameter update laws guarantee boundness of the parameters and avoid parameter drift. Another important remark is that no prior offline training is needed for tuning the online approximators. This makes the proposed online approximator unique and could be used for learning any unknown faults. In the next section, the performance of the fault detection scheme is examined mathematically.

#### IV. ANALYTICAL RESULTS

In this section, the selection of the threshold is derived mathematically. Next the stability and performance of the learning scheme (3) is derived mathematically by using the Lyapunov theory. To show the stability of the fault detection scheme, prior to the fault i.e.  $k \in [0, \mathbf{\infty})$ , under ideal conditions, the state and output estimation errors obtained from (1) and (2), are given by

$$\begin{aligned}e_s(k+1) &= Ae_s(k) - K(y(k) - \hat{y}(k)) - \hat{f}_s(y(k), u(k); \hat{\theta}_s(k)) \\ e_y(k) &= Ce_s(k) - \hat{f}_y(u(k); \hat{\theta}_y(k))\end{aligned}\quad (5)$$

Since  $\hat{\theta}_{0s}$  and  $\hat{\theta}_{0y}$  are chosen such that  $\hat{f}_s(y, u, \hat{\theta}_{0s}) = 0$  and  $\hat{f}_y(u, \hat{\theta}_{0y}) = 0$  for all  $y$  and  $u$ , the vectors  $(e_s, \hat{\theta}_{0s}) = (0, \hat{\theta}_{0s})$  and  $(e_y, \hat{\theta}_{0y}) = (0, \hat{\theta}_{0y})$  are the equilibrium points for the system in (5). Therefore,  $e_s(k) = 0$ ,  $e_y(k) = 0$ ,

$\hat{\theta}_s(k) = \hat{\theta}_{0s}$  and  $\hat{\theta}_y(k) = \hat{\theta}_{0y}$  for  $k \in [0, \mathbf{\infty})$ . Hence the errors converge to zero exponentially. Next for a non-ideal case or in the presence of modeling errors, the state and the output estimation error equations are given by

$$e_s(k+1) = Ae_s(k) + \eta_s(x(k), u(k), k) - Ke_y(k) - \hat{f}_s(y(k), u(k); \hat{\theta}_s(k)) \quad (6)$$

$$e_y(k) = Ce_s(k) + \eta_y(x(k), u(k), k) - \hat{f}_y(u(k); \hat{\theta}_y(k)) \quad (7)$$

To show the robustness of the scheme, we determine the appropriate value of the threshold, which would be the upper bound for  $e_y(k)$  under the no fault condition. Note prior to the fault, the online approximators in (6) and (7) are  $\hat{f}_s(y, u, \hat{\theta}_{0s}) = 0$  and  $\hat{f}_y(u, \hat{\theta}_{0y}) = 0$ . By solving (6) for  $e_s$  first and then (7) for  $e_y$ , we obtain the following

$$e_s(k) = \sum_{j=1}^k A_0^{k-j} (\eta_s(x(j-1), u(j-1), (j-1)) - K\eta_y(x(j-1), u(j-1), (j-1)))$$

$$\text{and } e_y(k) = C \sum_{j=1}^k (A_0^{k-j} (\eta_s(x(j-1), u(j-1), (j-1))$$

$$- K\eta_y(x(j-1), u(j-1), (j-1))) + \eta_y(x(j-1), u(j-1), (j-1)))$$

This in turn yields

$$\|e_y(k)\| \leq \left\| C \sum_{j=1}^k A_0^{k-j} \right\| \tilde{\eta}_s + \left\| C \sum_{j=1}^k A_0^{k-j} K \right\| \tilde{\eta}_y + \tilde{\eta}_y \quad (8)$$

which could be written as

$$\|e_y(k)\| \leq \mu_s \tilde{\eta}_s + (\mu_y + 1) \tilde{\eta}_y \quad (9)$$

$$\text{where } \mu_s := \left\| C \sum_{j=1}^k A_0^{k-j} \right\| = \left\| C \sum_{j=1}^{k=\infty} A_0^{k-j} \right\| \text{ for } k \in [0, \infty).$$

$$\text{Similarly } \mu_y := \left\| C \sum_{j=1}^k A_0^{k-j} K \right\| = \left\| C \sum_{j=1}^{k=\infty} A_0^{k-j} K \right\|. \text{ Thus if the}$$

size of the dead-zone is selected as  $\varepsilon = \mu_s \tilde{\eta}_s + (\mu_y + 1) \tilde{\eta}_y$ ,  $e_y(k)$  will remain within the dead zone for all  $k \leq \mathbf{\infty}$  and the output of the online approximator remains zero. Therefore, the adaptive scheme given by (2) is robust in the sense that it is not affected by modeling errors, provided their upper bounds are known beforehand.

Next after the fault at  $k \geq \mathbf{\infty}$ , the state and output estimation errors are respectively given by

$$\begin{aligned}e_s(k+1) &= Ae_s(k) + \eta_s(x(k), u(k), k) + \Pi_s(k - T_s) f_s(y(k), u(k)) \\ &\quad - Ke_y(k) - \hat{f}_s(y(k), u(k); \hat{\theta}_s(k))\end{aligned}$$

$$e_y(k) = Ce_s(k) + \eta_y(x(k), u(k), k) + \Pi_y(k - T_y) f_y(u(k)) - \hat{f}_y(u(k); \hat{\theta}_y(k)).$$

Next, the class of detectable faults occurring in the nonlinear discrete-time system is given by the following sensitivity theorem.

**Theorem 1 (Sensitivity):** *i) If there exist a time  $k_s > 0$ , such that  $f_s(y(k), u(k))$  satisfies*

$$\left| \sum_{i=T_s}^{T_s+k_s} CA_0^{(T_s+k_s-i)} \Pi_s(i-T_s) f_s(y(i), u(i)) \right| \geq 2\varepsilon,$$

Then the state fault will be detected i.e. the output error  $|e_y(T_s + k_s)| \geq \varepsilon$ .

ii) Also if there exists a time  $k_y > 0$  such that  $f_y(u(k))$  satisfies

$$\left| \Pi_y(k_y) f_y(u(T_y + k_y)) - \sum_{i=T_y}^{T_y+k_y} CA_0^{(T_y+k_y-i)} K \Pi_y(i-T_y) f_y(u(i)) \right| \geq 2\varepsilon$$

then the output fault will be detected i.e.  $|e_y(T_y + k_y)| \geq \varepsilon$ .

In the next theorem, the robustness of the fault detection scheme is examined. This theorem shows that the OLAD scheme does not adapt prior to the fault.

**Theorem 2 (Robustness):** The fault detection scheme in discrete-time ensures that the output of the online approximator (OLAD) remains zero prior to the occurrence of state or output fault for  $k \in [0, \mathbf{T})$  i.e.

$$\hat{f}_s(y(k), u(k); \hat{\theta}_{0s}(k)) = 0 \text{ and } \hat{f}_y(u(k); \hat{\theta}_{0y}(k)) = 0$$

**Proof:** Let us assume that for a finite time  $0 < k_1 < \mathbf{T}$

$|e_y(k)| < \varepsilon$  for  $k < k_1$  and

$$e_y(k_1) = \varepsilon \quad (10)$$

From the continuity of  $e_y(k)$  and the adaptive law (3), the parameters of the OLAD scheme will not be updated or adapted in the interval  $[0, k_1]$ . Hence in the interval  $[0, k_1]$  the error dynamics  $e_s(k)$  and  $e_y(k)$  satisfy

$$e_s(k+1) = A_0 e_s(k) + \eta_s(x(k), u(k), k) - K e_y(k)$$

$$e_y(k) = C e_s(k) + \eta_y(x(k), u(k), k)$$

The solution of  $e_y(k)$  is given as

$$\begin{aligned} |e_y(k_1)| &= \sum_{i=1}^{k_1} CA_0^{(k_1-i)} (\eta_s(x(i-1), u(i-1), i-1) \\ &\quad - K \eta_y(x(i-1), u(i-1), i-1)) + \eta_y(x(i-1), u(i-1), i-1) \\ &< \sum_{j=0}^{k_1} CA_0^{k-j} |\tilde{\eta}_s| + \sum_{j=0}^{k_1} CA_0^{k-j} K |\tilde{\eta}_y| = \mu_s \tilde{\eta}_s + (\mu_y + 1) \tilde{\eta}_y = \varepsilon \end{aligned}$$

Thus the above step contradicts our assumption in (10), which states that in the time interval  $k \in [0, \mathbf{T})$  the error  $e_y(k)$  remains within the threshold. Consequently, it can be deduced that the scheme is robust and the output of the online approximator remains zero prior to the fault.

The class of faults detectable by the proposed scheme in discrete-time was shown in Theorem 1 and the robustness of the scheme to uncertainties was analyzed in Theorem 2. For the class of detectable faults, it is always necessary to show

that the faults will be detected in a finite amount of time. To show that the detection time is finite, unlike in other schemes in discrete time [10-11], we propose the following theorem for the first time to determine the fault detection time explicitly. In this theorem, a method to calculate the detection time for both incipient and abrupt faults occurring in the state and output of the system is obtained. The detection time in continuous-time cannot be used here.

**Theorem 3 (Fault detection time):** The time taken to detect a state or output fault is obtained by solving the following:

i) **Incipient state fault**

Let us assume that the  $j$ 'th component of

$|f_{sj}(y(k), u(k))| \geq \beta_j$  for all  $k \in [T_s + k_i, T_s + k_d]$ , where

$\beta_j > 2\varepsilon_j$ , then the detection time for the incipient state fault could be obtained by solving the following equation

$$c_j \left( \frac{1 - a_{0j}^{k_d+1-k_i}}{1 - a_{0j}} \right) - c_j a_{0j}^{k_d} \left( \frac{\left( \frac{e^{-k_{sj}}}{a_{0j}} \right)^{k_i} - \left( \frac{e^{-k_{sj}}}{a_{0j}} \right)^{k_d+1}}{1 - \left( \frac{e^{-k_{sj}}}{a_{0j}} \right)} \right) = \left( \frac{2\varepsilon_j}{\beta_j} \right)$$

ii) **Abrupt state fault**

The detection time for abrupt state fault could be determined by solving the following equation

$$c_j \left( \frac{1 - a_{0j}^{k_d+1-k_i}}{1 - a_{0j}} \right) = \frac{2\varepsilon_j}{\beta_j}$$

iii) **Incipient output fault**

Let us assume that the  $j$ 'th component of  $|f_{yj}(u(k))| \geq \beta_j$  for

all  $k \in [T_y + k_i, T_y + k_d]$ , where  $\beta_j > 2\varepsilon_j$ , then the detection time for the incipient output fault could be obtained by solving the following equation

$$\begin{aligned} 1 - e^{-k_{sj} k_d} - c_j \left( \frac{1 - a_{0j}^{k_d+1-k_i}}{1 - a_{0j}} \right) \\ + c_j a_{0j}^{k_d} \left( \frac{\left( \frac{e^{-k_{sj}}}{a_{0j}} \right)^{k_i} - \left( \frac{e^{-k_{sj}}}{a_{0j}} \right)^{k_d+1}}{1 - \left( \frac{e^{-k_{sj}}}{a_{0j}} \right)} \right) = \left( \frac{2\varepsilon_j}{\beta_j} \right) \end{aligned}$$

iv) **Abrupt output fault**

In the case of an abrupt output fault, the detection time could be determined by solving the equation

$$1 - c_j \left( \frac{1 - a_{0j}^{k_d+1-k_i}}{1 - a_{0j}} \right) = \frac{2\varepsilon_j}{\beta_j}$$

where  $j=1, 2, \dots, n$

Hence for faults in multiple states, we find the fault detection time as  $k_{dt} = \min(k_{dj})$ ,  $j = 1, \dots, n$ . Next we proceed to show the stability and the performance of the fault detection scheme. In the proposed fault detection scheme, the OLAD had to be tuned online, which is accomplished by using the parameter update law in (3). The following theorem guarantees the

stability of the proposed fault detection scheme. In the earlier works [10-11], the PE condition had to be satisfied to ensure a stable performance of the scheme. But the following theorem is proposed as an improved parameter tuning schemes for the fault detection so that the PE is relaxed and also is applicable for the non-ideal case i.e. system with unmodeled dynamics.

**Theorem 4 (Stability):** (PE condition not required) let the initial conditions for the nonlinear estimator considered to be bounded in a region  $U \subset \mathfrak{R}^n$ . In the presence of modeling and approximation errors, consider the parameter update laws given in (3). Then the state, output and parameter estimation errors  $e_s(k)$ ,  $e_y(k)$ ,  $\hat{\theta}_s(k)$  and  $\hat{\theta}_y(k)$  respectively are uniformly ultimately bounded (UUB) provided the design parameters are selected as  $\alpha_s \|Z_s\|^2 < 1$ ,  $0 < \gamma_s < 1$ ,  $\alpha_y \|Z_y\|^2 < 1$ ,  $\lambda_{a_{\max}}(A_0) < 1$  and  $0 < \gamma_y < 1$ .

To illustrate further the performance of the fault detection scheme, the developed fault detection scheme is tested on a continuous stir tank reactor (CSTR). The simulation results show the performance of the fault detection scheme as discussed in detail next.

## V. SIMULATION RESULTS

The discrete time state space model of a CSTR is given below [9]

$$\begin{aligned} x_1(k+1) &= t_s \left[ -y_1(k) + d(1 - y_1(k)) \exp\left(\frac{y_2(k)}{c + y_2(k)/\delta}\right) \right] + x_1(k) \\ x_2(k+1) &= t_s \left[ -y_2(k)(1 + \zeta) + Ho.d(1 - y_1(k)) \exp\left(\frac{y_2(k)}{c + y_2(k)/\delta}\right) + F(k) \right] + x_2(k) \\ y_1(k+1) &= x_1(k+1) \text{ and } y_2(k+1) = c.x_2(k+1) \end{aligned} \quad (11)$$

where  $x_1$  and  $x_2$  are the states of the system,  $y_1$  and  $y_2$  are the outputs. This CSTR system is simulated with state and output faults. Following are the values of the system parameters chosen for the following two simulations,  $\delta = 13.4$ ,  $d = 1.0$ ,  $\zeta = 0.5$ ,  $c = 0.75$ ,  $t_s = 0.15$ , and  $Ho = 2.5$ . As a first case, the system is simulated with a state fault as follows.

### i) State fault

We assume an incipient state fault with the time profile

$$\Pi_s(k - T_s)f_s(y(k)) = 0.5 \left( 1 - e^{-\kappa_{s2}(k - T_s)} \right) \begin{bmatrix} 0 \\ -(y_2(k)/0.75)^4 \end{bmatrix} \quad (12)$$

where the state fault grows at a rate of  $\kappa_{s2} = 0.1$ . The following nonlinear estimator is used to study the system described in (11) with the fault given by (12)

$$\begin{aligned} \hat{x}_1(k+1) &= t_s \left[ -y_1(k) + d(1 - y_1(k)) \exp\left(\frac{y_2(k)}{c + y_2(k)/\delta}\right) \right] \\ &\quad + a_1 \hat{x}_1(k) + K_1(y_1(k) - a_2 \hat{y}_1(k)) \end{aligned}$$

$$\begin{aligned} \hat{x}_2(k+1) &= t_s \left[ -y_2(k)(1 + \zeta) + Ho.d(1 - y_1(k)) \exp\left(\frac{y_2(k)}{c + y_2(k)/\delta}\right) \right] \\ &\quad + F(k) - \hat{f}_s(y(k), \hat{\theta}_s(k)) \\ &\quad + a_3 \hat{x}_2(k) + K_2(y_2(k) - \hat{y}_2(k)) \end{aligned} \quad (13)$$

where  $\hat{x}_1$ ,  $\hat{x}_2$  are estimated states and  $\hat{y}_1$  and  $\hat{y}_2$  the estimated output of the system in (13). The values of the design constant are chosen as  $K_1 = 1.0$ ,  $K_2 = 1.9$ ,  $a_1 = 0.5$ ,  $a_2 = 0.5$ , and  $a_3 = 0.9975$ . The input to the system (12) and the nonlinear estimator (13) is given by  $F(k) = 0.5 \sin(k)$ . The online approximator  $\hat{f}_s(y(k), \hat{\theta}_s(k))$  is a radial basis function network and is tuned online using the parameter update law (3). The NN input is taken as  $y(k)$  and the output is given by

$$\hat{f}_s(y, \hat{\theta}_s) = \sum_{j=1}^N \hat{\theta}_{sj} \exp\left(\frac{-|y - c_{sj}|^2}{\sigma_s^2}\right)$$

where  $N$  is the number of nodes,  $c_{sj}$ ,  $j = 1, 2, \dots, N$  are centers,  $\sigma$  is the width of the basis function of the network, and  $\hat{\theta}_{sj}$ ,  $j = 1, 2, \dots, N$  are parameters of the network.

For this simulation, the online approximator is a single layer radial basis function network with  $N=8$  neurons, and the initial parameter values of the network ( $\hat{\theta}_s$ ) is chosen randomly. The centers ( $c_s$ ) of the network are chosen randomly in the interval  $[-10, 10]$ . The width of the radial basis function is taken as  $\sigma_s = 0.6$ . The fault occurs at  $T_s = 15$  seconds. The learning rate and the design parameter of the proposed parameter update for  $\hat{\theta}_s(k)$  in (3) are  $\alpha_s = 0.01$  and  $\gamma_s = 0.9$  respectively.

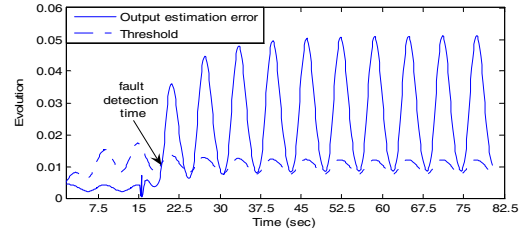


Fig. 1: Output estimation error and detection threshold.

Fig. 1 depicts the output estimation error and a fault is detected when this error exceeds the threshold. For this simulation, we assume a 10% uncertainty in the value of  $\zeta$ ; hence the modeling uncertainty is given by  $|\eta_s(x(k), k)| = |0.05x_2(k)|$ . Also assuming no uncertainty in measurement or noise  $\eta_y(x(k), u(k), k) = 0$ , we derive the

threshold as  $\lambda_s \cdot (1 - \mu_s / 1 - \mu_s) \cdot \eta_s(x(k), k)$ . By choosing  $\lambda_s = 0.045$ ,  $\mu_s = 0.8$ , we define a time varying threshold as  $0.0045 * \left( \frac{1 - 0.8^k}{1 - 0.8} \right) * 0.05x_2(k)$ . The plot of the threshold is also illustrated in Fig. 1. From this figure, the fault is detected at  $k_d = 19.35$  sec. The fault detection time is also shown in Fig. 1 which appears that this fault is detected in 4.35 seconds after it has occurred.

The single-layer based OLAD scheme is able to approximate the nonlinear dynamics except during sudden changes.

## ii) Output fault

Next an output fault defined by

$$\Pi_y(k - T_y) f_y(u(k)) = 1 - e^{-\kappa_{y1}(k - T_y)} \cdot \begin{bmatrix} 1000 \cdot (0.1 \sin(u(k)))^2 \\ 0 \end{bmatrix} \quad (14)$$

is induced where  $u(k) = F(k)$  and the output fault growth rate is given by  $\kappa_{y1} = 0.001$ . The system (11) with a fault given in (14) is studied using the nonlinear estimator (13) with the design parameters as follows

$K_1 = 0.95$ ,  $K_2 = 1.3$ ,  $a_1 = 0.95$ ,  $a_2 = 1$ , and  $a_3 = 0.975$ .

The OLAD scheme,  $\hat{f}_y(u, \hat{\theta}_y)$ , used for learning the output fault,  $f_y(u)$ , is a single layer radial basis function network with  $N=8$  neurons. The input to the OLAD scheme is  $u(k)$  and it could mathematically be expressed as

$$\hat{f}_y(u, \hat{\theta}_y) = \sum_{j=1}^{N=8} \hat{\theta}_{yj} \exp \left( \frac{-|u - c_{yj}|^2}{\sigma_y^2} \right)$$

The initial values of the network parameter ( $\hat{\theta}_y$ ) is chosen randomly and the centers ( $c_y$ ) of the network are chosen randomly in the interval  $[-15, 15]$ . The width of the radial basis function is taken to be  $\sigma_y = 0.5$ . The fault (14) occurs at  $T_y = 15$  sec and the proposed weight update law for  $\hat{\theta}_y(k)$  in (3) is used for tuning the parameters of the OLAD scheme with the learning rate taken as  $\alpha_y = 0.01$  and  $\gamma_y = 0.1$ .

In Fig. 2, the output estimation error and the thresholds are displayed. For this simulation, a fixed modeling uncertainty i.e.  $\eta_y(x(k), u(k), k) = 0.2$  was selected and therefore a constant

threshold  $0.05 * \left( \frac{1}{1 - 0.85} \right) * 0.2 = 0.0667$  is obtained. This figure illustrates the estimation error and detection time. In this case, the fault is found to occur at 16.65 sec. whereas the fault is detected within 1.65 sec after its occurrence thus enabling us to detect the fault as quickly as possible to avoid any catastrophic failures.

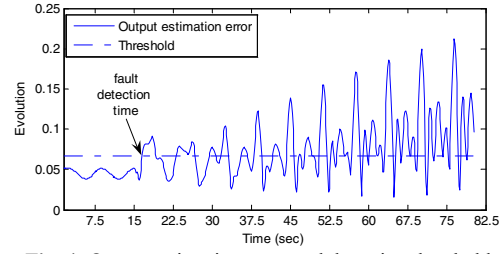


Fig. 4: Output estimation error and detection threshold.

Although the simulation results are shown for incipient faults, abrupt state or output faults can also be detected. The performance of the fault detection scheme is highly satisfactory in learning the unknown fault dynamics.

## VI. CONCLUSIONS AND FUTURE WORK

A robust model based fault detection scheme for nonlinear discrete time multivariable system was developed and tested in this paper. Through rigorous analytical proofs and simulation results, the stability and performance of the fault detection scheme was demonstrated. The future work is to test the proposed fault detection scheme in a real-time example.

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